Priors and Bayesian Inference. To complete the Bayesian model specifications, priors have to be assigned. For the direct effects $\beta_j$, we assume diffuse priors

$$p(\beta_j) \propto \text{const}, \quad p(\beta_j) \propto \text{const}, \quad p(\alpha) \propto \text{const}.$$ 

For the factor loadings, we specify informative Gaussian priors

$$p(\lambda_j) \propto N(0, \sigma_j^2),$$

with $\sigma_j^2 = 1$ as the standard choice, to avoid the so-called Heywood cases (see e.g. Heywood). 

Priors for functions. For a function $f(w)$ of a continuous covariate $w$, we assume Bayesian P-spline-priors as in Brezger and Lang (2005). More specifically, the $f(w)$ approximated by a spline function and represented as a linear combination

$$f(w) = \Sigma_{i=1}^{L} \alpha_i B_i(w),$$

where $B_i(w)$ are B-spline basis functions. The vector $\alpha = (\alpha_1, ..., \alpha_L)$ follows a Gaussian random walk model of first or second orders, i.e the prior is Gaussian with
Priors for spatial effects. We usually split $f_{\text{geo}}(s)$ into a smooth structured effect and an unstructured effect, i.e.

$$f_{\text{geo}}(s) = f_{\text{str}}(s) + f_{\text{unstr}}(s)$$

where $f_{\text{str}}(s)$ models global spatial trends while $f_{\text{unstr}}(s)$ captures local effects. For $f_{\text{str}}(s)$ we assume a Markov random field prior (Beseg, York and Mallie, 1990). The spatial smoothness prior of function evaluations $f_{\text{str}}(s)$ is

$$f_{\text{str},s} | f_{\text{str},t}, t \neq s, \tau^2_{\text{str}} \sim N \left( \sum_{t \in \delta_s} \frac{f_{\text{str},t}}{N_s}, \frac{\tau^2_{\text{str}}}{N_s} \right),$$

(13)

where $N_s$ is the number of adjacent sites and $t \in \delta_s$ denotes, that site $f_s$ is a neighbor of site $f_t$. Thus the (conditional) mean of $f_s$ is an unweighted average of function evaluations of neighboring sites. Note that for spatial data conditioning is undirected since there is no natural ordering of different sites $f_s$ as in the case for metrical covariates.

For the uncorrelated effect, we assume i.i.d. Gaussian random effects, i.e.

$$f_{\text{unstr}}(s) \sim N(0, \tau^2_{\text{unstr}}) \quad s = 1, \ldots, S$$

The variance parameter $\tau^2$ controls the amount of the smoothness. It is estimated automatically by the inverse gamma $IG(a,b)$ with known hyperparameters $a$ and $b$. Standard choices for the hyperparameters are $a = 1$ and $b = 0.005$ or $a = b = 0.001$. 

\[
p(\alpha) \propto \exp\left(-\frac{1}{2\tau^2} \alpha'K\alpha\right)
\]